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ABSTRACT

Predicted fission cross section values, based on systematic trends observed recently by Behrens, are presented in the neutron energy interval from 3 to 5 MeV for forty-three isotopes, ranging from thorium through californium. Comparisons with measured fission cross section values are included for thirty-three isotopes.

INTRODUCTION

The use of fission systematics to predict cross sections can be traced to Huizenga[1] (1953) and Batzel[2] (1954) who correlated σ_f/σ_T and Γ_n/Γ_f ratios with the fissionability parameter Z^2/A. In 1955, ?irschall[3] showed an apparent correlation between measured fission cross sections at an incident-neutron energy of 3 MeV a.?d the parameter $Z^{4/3}/A$; how ?er, no significance to this parameter was claimed. Smith et al.[4] later showed that for twelve isotopes, ranging from radium through plutonium, the

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neutron-induced fission cross sections averaged over the energy range 2 to 5.5 MeV (on the first fission plateau) varied linearly with the empirical function $[Z^2/(A + 1)^{3/2}]^n$ where n could vary from -2 to +4. This function fit the data available at that time within the experimental uncertainties. The parameter $Z^{4/3}/A$ was consistent with this scheme; however, Z^2/A was not. Howerton[5] observed that fission cross sections for the isotopes of each element follow a definite, but nonlinear, relationship as a function of $Z^{4/3}/A$. Similar studies and discussions of fission systematics were presented in Refs. 6-9.

METHODOLOGY

Recently, Behrens[10] observed that measured fission cross section ratios[11-14] on the first fission plateau over the atomic mass range A=230 to 252 and element range Z = 90 to 98 exhibit straightforward systematic behavior as a function of both constant proton number and constant neutron number. For constant neutron number, fission cross sections in the neutron energy interval 3 to 5 MeV tend to increase linearly with Z. Figure 1 shows this trend for isotopes with N = 140 through 149. For the lines labeled N = 142, 144, 146, 148, the recent measurements of References 11, 13, 14 were used to determine the equations of the lines even though there were only two measurements for each value of N. This was for two reasons: the experimental uncertainties are generally much less for these measurements than for those done earlier and, secondly, the measurements reported in References 11, 13, 14 were all done with the same technique and at the same institution. For the line labeled 140, a least squares fit of the data of Refs. 17, 18, and 20, was done to obtain the equation of the line.

Each of the five lines can be expressed as:

$$A = m(N) \times \sigma_R(Z,A) + B(N) \tag{1}$$

A more usual representation with σ_R being the dependent variable and A being the independent variable results in physically unmeaningful values for the intercepts of the lines on the ordinate. It also results in much larger residuals when values of the slopes and intercepts are fit as polynomials in N because of the relatively small range in the independent variable for which data points are available. We have, therefore, chosen to find representations for m(N) and B(N) for equation (1) by least squares fitting of the five values of these quantities obtained from the solid lines shown in Figure 1. Equation (1) can be rewritten as:

$$\sigma_R(Z,A) = \frac{A - B(N)}{m(N)} \quad , \tag{2}$$

where

$$B(N) = \sum_{i=0}^{3} \beta_i N^i \quad , \tag{3}$$

and

$$m(N) = \sum_{i=0}^{3} \alpha_i N^i \quad . \tag{4}$$

Fits to obtain first and second order representations were discarded after testing against experimental values. The values obtained for the coefficients

α_i, β_i are given in Table I. From Equation (2) values for the ratio of the fission cross section of an isotope with mass A and charge Z to the fission cross section of ^{235}U in the 3 to 5 MeV range can be calculated. We obtained cross sections from Equation (2) by multiplying the calculated ratios by 1.16 barns, the value for the average of the ^{235}U fission cross section from 3 to 5 MeV.[15,16] In Table II, the predicted and measured fission cross sections are compared for the thirty-three isotopes that have measured cross sections reported in or near the 3 to 5 MeV energy range. Because of the paucity of data, we have included many single point measurements with energies as low as 1.5 MeV. For the convenience of the reader we have included in Table II the predicted fission cross sections for 10 isotopes that have not been measured, but are of interest for one or more reactor programs.

DISCUSSION

With the exception of ^{233}Th, the agreement between predicted and measured fission cross sections is apparently acceptable. Quite clearly, the negative cross section obtained for ^{233}Th using Eq. (2) is ridiculous, but ^{233}Th has the smallest fission cross section of all the isotopes for which measurements are reported. In the absence of measured fission cross sections (and only then) the predicted values from Eq.(2) are useful for applications such as the trans-plutonium isotope production problem for plutonium-based breeder reactors or the trans-thorium isotope production problem for thorium-based breeder reactors.

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TABLE CAPTIONS

Table I

Coefficients for Eqs. (3) and (4) obtained by least-squares fitting of the slopes and intercepts of the five lines of constant neutron number (N) shown in Fig. 1 as solid lines.

Table II

Comparison of measured fission cross sections with predicted values obtained from ratios calculated using Eq. (2) multiplied by 1.16 barns (the average fission cross section of ^{235}U in the 3 to 5 MeV energy range).

TABLE I

Coefficient	Value
α_o	$-1.21315882 \times 10^{4}$
α_1	2.51795703×10^{2}
α_2	$-1.74231696 \times 10^{0}$
α_3	$4.02000000 \times 10^{-3}$
β_o	1.96175369×10^{4}
β_1	$-4.06932552 \times 10^{2}$
β_2	2.83841428×10^{0}
β_3	$-6.57812500 \times 10^{-3}$

Nuclide	Measured $\sigma(n,f)$ (barns)	E_n (MeV)	Year	Ref.	Predicted σ (n,f) (barns)	Pred./Meas.
Th-230*	0.41 ± 0.08	2.5	1960	17	0.43	1.05
Th-231	0.21 ± 0.03	2.0	1970	18	0.28	1.33
Th-232*	0.147 ± 0.003	3-5	1977	14	0.11	0.75
Th-233	0.12 ± 0.02	2.0	1970	18	-0.07	-0.60
Pa-231*	1.47 ± 0.19	3.0	1971	19	1.34	0.91
Pa-232					1.03	--
Pa-233					0.78	--
U-232*	2.2 ± 0.3	1.5	1971	20	2.25	1.02
U-233	1.70 ± 0.03	3-5	1975	12	1.78	1.05
U-234*	1.45 ± 0.01	3-5	1977	11	1.44	0.99
U-235	1.16 ± 0.02	3-5	1975,1976	15,16	1.17	1.01
U-236*	0.905 ± 0.008	3-5	1977	11	.93	1.03
U-237	0.63 ± 0.09	2.25	1970	18	0.71	1.13
U-238*	0.560 ± 0.04	3-5	1977	11	0.54	0.96
U-239	0.52 ± 0.08	2.25	1970	18	0.42	0.81
U-240					0.36	--
Np-237*	1.52 ± 0.02	3-5	1977	14	1.52	1.00
Np-238					1.28	--
Np-239					1.08	--
Pu-236					2.78	--

* Isotope used for fitting procedure.

TABLE II (continued)

Nuclide	Measured σ(n,f) (barns)	E_n (MeV)	Year	Ref.	Predicted σ(n,f) (barns)	Pred./Meas.
Pu-237					2.41	--
Pu-238	2.23 ± 0.11	3.0	1967	21	2.11	0.95
Pu-239	1.78 ± 0.02	3-5	1976	13	1.85	1.04
Pu-240*	1.62 ± 0.03	3-5	1976	13	1.62	1.00
Pu-241	1.43 ± 0.02	3-5	1976	13	1.44	1.01
Pu 242*	1.30 ± 0.02	3-5	1976	13	1.30	1.00
Pu-243	1.31 ± 0.20	2.0	1970	18	1.22	0.93
Pu-244	1.11 ± 0.02	3-5	1976	13	1.18	1.06
Am-241	2.04 ± 0.05	3-5	1977	14	2.16	1.06
Am-242m	2.0 ± 0.3	2.5	1968	22	1.95	0.98
Am-243*	1.77 ± 0.04	3-5	1977	14	1.77	1.00
Cm-242					2.70	--
Cm-243	2.3 ± 0.3	3.0	1976	23	2.46	1.07
Cm-244	1.92 ± 0.12	2.1	1971	24	2.24	1.17
Cm-245	2.15 ± 0.25	2.1-2.5	1971	24	2.06	0.96
Cm-246	1.8 ± 0.2	2.3-2.8	1971	24	1.92	1.07
Cm-247	2.8 ± 0.7	1.9	1971	24	1.83	0.65
Cm-248	1.75 ± 0.15	2.3-2.8	1971	24	1.77	1.01
Bk-249	1.58 ± 0.10	3.0	1977	25	2.03	1.28
Cf-249	1.9 ± 0.2	2-3	1973	26	2.46	1.29
Cf-250					2.30	--
Cf-251					2.18	--
Cf-252	2.5 ± 0.3	3-5	1971	27	2.09	0.84

FIGURE CAPTION

Figure 1

Experimental ratios of fission cross sections to the ^{235}U fission cross section in the 3 to 5 MeV energy range plotted against the atomic mass of the target nucleus (A). The open circles (Refs. 11, 13, and 14) and closed circles (Refs. 17, 18, and 20) were used for obtaining slopes and intercepts of the lines of constant neutron number (N = 140, 142, 144, 146, 148). The triangles are measurements that were not used for deriving the constants of Table 1. Error bars are shown where the error is larger than the symbol.

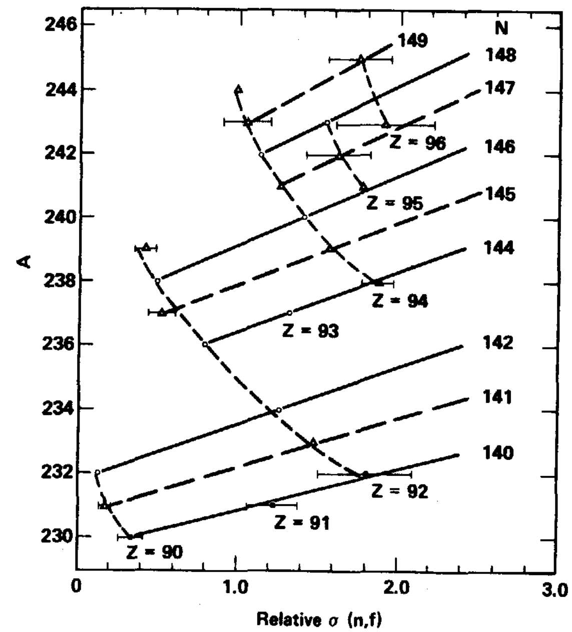

FIGURE 1